CONDUCTIVITY OF QUANTUM WELLS: ROLE OF ELECTRON-ACOUSTIC PHONON INTERACTION

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Abstract

Conductivity is a crucial and widely recognized concept in material science, particularly significant in the study of low-dimensional systems. This research extends the analysis of the conductivity tensor within a quantum well with infinite potential, focusing on electron-acoustic phonon scattering. The system is subjected to two external fields: an electromagnetic wave and a laser field. The study explores the detailed effects of these external fields, noting that significant impacts occur only at high frequencies. Among the factors affecting conductivity, the amplitude of the laser field is the most influential. Additionally, when the electromagnetic wave frequency exceeds 10^{12} s^{-1}, its impact on conductivity becomes considerable.

Keywords: acoustic phonon, conductivity tensor, electromagnetic wave, laser field, quantum well

1. Introduction

Low-dimensional semiconductor systems have emerged as a pivotal research direction in condensed matter physics and materials science, offering transformative potential for both fundamental studies and technological applications. The miniaturization of semiconductors down to nanometric scales has enabled the realization of quantum confinement effects, which give rise to distinct quantum phenomena not observable in bulk materials. As a consequence, semiconductor structures such as quantum wells (2D), quantum wires (1D), and quantum dots (0D) have become essential platforms for exploring the interplay between quantum mechanics and material properties (Nguyen Quang Bau et al., 2012; Bui Dinh Hung et al., 2012; Nguyen Quoc Anh et al., 1995).

Over the past few decades, sustained research into these low-dimensional systems has led to remarkable advances, culminating in a new class of electronic and optoelectronic devices with superior performance in terms of speed, integration density, and energy efficiency (Balandinand Wang, 1998; Khokhlov et al., 2000; Kryuchkov et al., 2008; Grinberg and Luryi, 1988; Butscher and Knorr, 2006; Blencowe and Skik, 1996). Among these, the quantum well stands out as a prototypical two-dimensional (2D) system, where electrons are confined in one spatial direction—typically by a potential well—while remaining free in the other two. This spatial confinement quantizes the energy spectrum of the carriers along the confined direction, fundamentally altering the electronic density of states and enabling tunability of optical and electronic properties through engineering of the potential landscape (Grinberg and Luryi, 1988; Butscher and Knorr, 2006; Blencowe and Skik, 1996).

The specific energy levels and corresponding wavefunctions in a quantum well depend sensitively on the geometry and height of the confining potential. While various models—such as finite or triangular wells—have been extensively studied, the infinite potential well remains an important idealization, serving as a starting point for analytical investigation and a benchmark for more complex scenarios (Boiko et al., 1993; Geyler et al., 2000; Hoang Van Ngoc et al., 2017; Nguyen Quang Bau et al., 2009, 2010; Hoang Dinh Trien et al., 2011). In this work, we focus on quantum wells with infinitely high potential barriers, allowing for precise analysis of quantized electronic states and their response to external perturbations.

A central quantity in evaluating the electronic transport behavior of materials is the electrical conductivity. Defined via the relationship between current density and electric field, conductivity encapsulates a material's ability to transport charge carriers under an applied field. For semiconductors, the conductivity typically spans a wide range—from fractions to several S/m—significantly lower than that of metals due to lower free carrier densities and higher scattering rates (Nguyen Quoc Anh et al., 1995). Nonetheless, conductivity in semiconductors is highly tunable via doping, temperature, and external fields, rendering it a key focus in the study of transport phenomena.

External fields, particularly electromagnetic and laser fields, can exert profound influences on the carrier dynamics in semiconductor systems. The interaction between these fields and the charge carriers depends on factors such as field intensity, frequency (or wavelength), and polarization. Shorter-wavelength fields can penetrate deeper into the atomic structure and influence inner or core-level electrons, while fields of moderate energy—such as infrared or visible laser light—primarily affect conduction electrons in the outer shell. These interactions modulate carrier scattering mechanisms, effective mass, and energy dispersion, ultimately modifying the conductivity of the system (Geyler et al., 2000).

In this study, we investigate the behavior of a quantum well subjected to two types of external fields: a polarized electromagnetic wave and a laser field. Both fields are chosen to have moderate photon energies to ensure they predominantly couple with conduction electrons, which are primarily responsible for charge transport. Furthermore, we limit our focus to electron–acoustic phonon interactions, which are dominant at low to moderate temperatures and significantly contribute to the resistive behavior in semiconductors. By analyzing how external fields modulate these interactions, we aim to elucidate their effect on the conductivity of quantum wells with infinite potential confinement.

2. Conductivity tensor

Suppose, considering a quantum well with thickness d, the energy system ($\hbar = 1$):

$$\varepsilon_{n,p_\perp} = \frac{p_\perp^2}{2m} + \frac{\pi^2}{2md^2} n^2 \tag{1}$$

$n = 0,1,2,\ldots$; $p_\perp^2 = p_{x0y}^2 = p^2 - p_z^2$; $\vec{p}$ is momentum; two external field:

$$\begin{cases} \vec{E}_t = \vec{E}e^{-i\omega t} + \vec{E}e^{i\omega t} \\ \vec{H} = [\vec{n}, \vec{E}_t] \end{cases}, \text{ and } \vec{F}_t = \vec{F}\sin(\Omega t) \tag{2}$$

withω and Ω arethe frequency of electromagnetic field and laser field, respectively.

The quantum kinetic equation for this system is ($\hbar = 1$) (Nguyen Quang Bau et al., 2012):

$$\frac{\partial f_{n,t}(\vec{p}_\perp)}{\partial t} + \left(e\vec{E}_t + \omega_H\left[\vec{p}_\perp, \vec{h}_t\right], \frac{\partial f_{n,t}(\vec{p}_\perp)}{\partial \vec{p}_\perp}\right) = \\ = 2\pi \sum_{n,n',\vec{q}} M_{n,n',\vec{q}} \sum_{l=-\infty}^{+\infty} J_l^2(\vec{a},\vec{q})\left[f_{n',t}(\vec{p}_\perp + \vec{q}) - f_{n,t}(\vec{p}_\perp)\right]\delta\left(\varepsilon_{n',\vec{p}_\perp+\vec{q}} - l\Omega - \varepsilon_{n,\vec{p}_\perp}\right) \tag{3}$$

$f_n(\vec{p}_\perp, t)$ is function ofdistribution;

$$M_{n.n',\vec{q}} = \left|C_{\vec{q}}\right|^2 I_{n,n'}^2 N_{\vec{q}} \tag{4}$$

$\vec{q}$ is momentum of phonon;

In case of scattering between electrons and acoustic phonons:

$$N_q = \frac{k_B T}{v_s q_\perp};\ C_q^2 = \frac{\xi^2 q_\perp}{2\rho v_s V}; \tag{5}$$

When t = 0 (G. M. Shmelev et al., 1982):

$$\vec{j}(t=0) = \int(\vec{R}(\varepsilon) + \vec{R}^*(\varepsilon))d\varepsilon = \vec{j}_1 + \vec{j}_1^*$$

$$= \frac{4e^2 n_0}{m} \frac{\tau(\varepsilon_F)}{1+\omega^2\tau^2(\varepsilon_F)}\left\{\left[\varepsilon_F - \frac{\pi^2}{2md^2}n^2\right]I + \lambda\frac{\tau(\Omega)\left[1-\omega^2\tau(\Omega)\tau(\varepsilon_F)\right]}{1+\omega^2\tau^2(\Omega)} + \\ - A\frac{\tau(\varepsilon_F)\left[1-\omega^2\tau^2(\varepsilon_F)\right]}{1+\omega^2\tau^2(\varepsilon_F)}\right\}\vec{E} \tag{6}$$

$$A = \frac{e^2F^2}{2m\Omega^3} M_{n,n'}(2m\Omega)\sqrt{2m\left(\varepsilon_F - \frac{\pi^2}{2md^2}n^2\right)} \tag{7}$$

$$\lambda = \frac{e^2F^2}{2m\Omega^3} M_{n,n'}(2m\Omega)\left\{\sqrt{2m\left(\varepsilon_F - \frac{\pi^2}{2md^2}n^2\right)}\left[\sqrt{2m\left(\Omega - \frac{\pi^2}{2md^2}n^2\right)} - 1\right]\right\} \tag{8}$$

$$M_{n.n'}(2m\Omega) = \left|C_{\vec{q}}\right|^2 I_{n,n'}^2 N_{\vec{q}} = \frac{\xi^2}{2\rho v_s V} k_B T \left|I_{n,n'}\right|^2 \tag{9}$$

Because $\vec{j}(t=0) = \sigma\vec{E}(t=0) = \sigma 2\vec{E}$ [3]so:

$$\sigma_{ik} = \frac{2e^2 n_0}{m}\frac{\tau(\varepsilon_F)}{1+\omega^2\tau^2(\varepsilon_F)}\left\{\left[\varepsilon_F - \frac{\pi^2}{2md^2}n^2\right]\delta_{ij} + \frac{\tau(\Omega)\left[1-\omega^2\tau(\Omega)\tau(\varepsilon_F)\right]}{1+\omega^2\tau^2(\Omega)}\lambda + \\ - \frac{\tau(\varepsilon_F)\left[1-\omega^2\tau^2(\varepsilon_F)\right]}{1+\omega^2\tau^2(\varepsilon_F)}A\right\} \tag{10}$$

Expression (10) represents the analytical formulation of the electrical conductivity for the quantum well system under investigation. This expression explicitly encapsulates the influence of external electromagnetic and laser fields on the charge transport properties. The functional dependence of the conductivity on the external fields reflects modifications to both the energy dispersion and carrier scattering mechanisms induced by field–electron interactions.

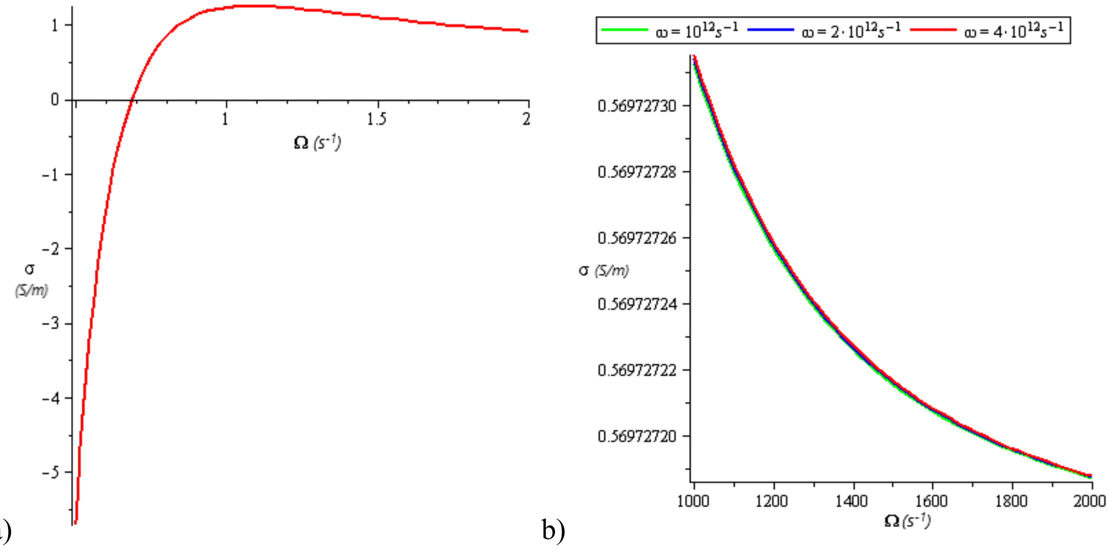
A detailed inspection of the expression reveals that the conductivity is governed by a complex interplay of multiple parameters. These include intrinsic parameters of the quantum well—such as well width, effective mass of the carriers, and quantized energy levels—alongside characteristics of the confining potential. In the case of infinite potential barriers, the wavefunctions adopt a well-defined sinusoidal form, and the energy levels scale quadratically with the quantum number. These features directly influence the density of states and transition rates, which in turn shape the transport behavior.

Moreover, the conductivity expression incorporates parameters that characterize the interaction between electrons and acoustic phonons. These include the deformation potential constant, phonon velocity, and temperature-dependent phonon occupation number. The scattering of charge carriers by acoustic phonons is a primary mechanism of momentum relaxation, especially in low-dimensional systems where confinement enhances electron–phonon coupling due to restricted phase space and modified screening.

Notably, the external fields modify the transition matrix elements involved in the electron–phonon interaction, introducing additional field-dependent terms in the scattering rate. This results in either enhancement or suppression of conductivity, depending on the field configuration and resonance conditions. Hence, the derived expression provides a theoretical framework to quantify how conductivity varies with the strength, frequency, and polarization of the applied fields, offering valuable insights into the tunability of transport properties in quantum-confined semiconductor structures.

3. Discussion

Here we plot for the caser of the quantum well GaAs/GaAsAl with $\tau(\varepsilon_F) \sim 10^{-11}$ s^{-1}; $v_s = 5220 m/s$; $n_0 = 10^{23} m^{-3}$; $\rho = 5.3\times10^3 kg/m^3$; $\xi = 2.2\times10^{-8} J$; $d = 100\ \overset{0}{A}$; $T = 300K$.



a) b)

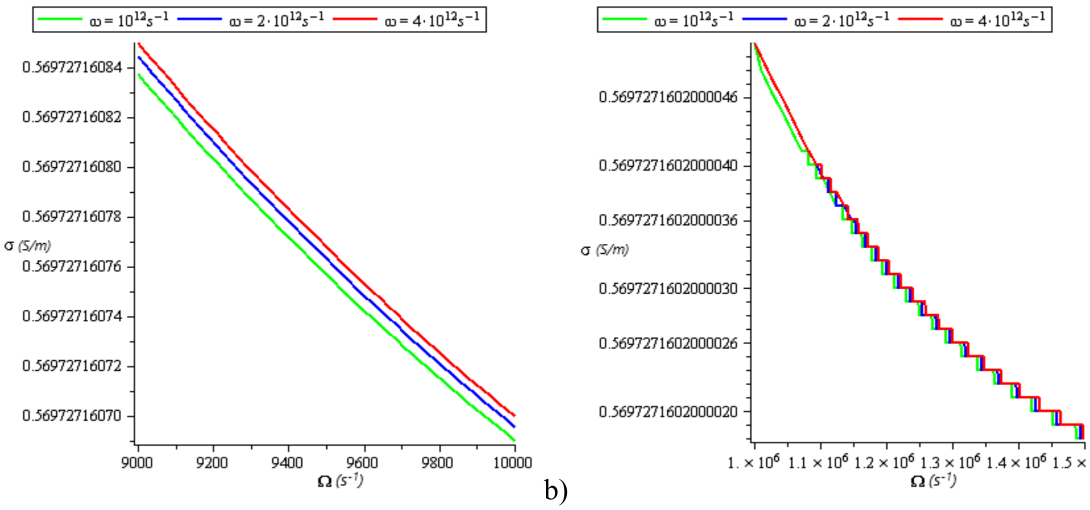
Figure 1. The variation of σ on Ω when Ω is very small (a) and $10^3 < \Omega < 2.10^3$ (b).

Figure 1a illustrates the behavior of the system's electrical conductivity as a function of laser field frequency in the low-frequency regime. In this range, corresponding to frequencies approaching zero, the observed variation in conductivity is primarily of qualitative interest. From a physical standpoint, these extremely low-frequency values are not characteristic of practical laser fields, and thus the insights drawn from this region serve mainly to establish a baseline for comparative purposes.

As the laser frequency increases into the range of approximately $10^3\ s^{-1}$ to $2*10^3\ s^{-1}$, shown in Figure 1b, the conductivity exhibits a mild parabolic decline. Importantly, the conductivity curves corresponding to three different electromagnetic wave frequencies are nearly indistinguishable in this regime. This convergence suggests that the contribution of the laser field to electron dynamics remains negligible at such low photon energies. Physically, this can be attributed to the fact that low-energy photons are insufficient to induce significant perturbations in the energy states or mobility of conduction electrons within the quantum well structure.

In contrast, when the laser frequency is increased to the intermediate range of $9*10^3\ s^{-1}$ to $10^4\ s^{-1}$, as depicted in Figure 2a, the conductivity curves begin to diverge. This divergence marks the onset of noticeable interaction between the laser field and the electronic subsystem. At these frequencies, photon energies become comparable to the energy differences between quantized levels or to the energy scale of phonon scattering processes, thus enabling more efficient coupling. The resulting variation in conductivity reflects the emergence of laser-induced modifications to the electron mobility, including changes in scattering rates or effective mass renormalization.

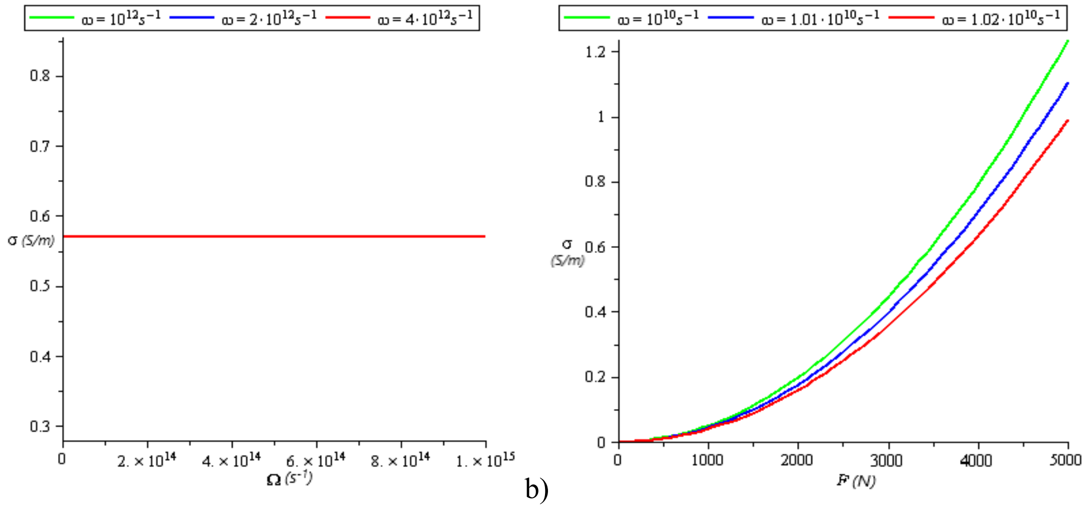
A further increase in frequency, from $10^6\ s^{-1}$ to $1.5*10^6\ s^{-1}$, presented in Figure 2b, reveals a continued overall decline in conductivity. However, the three conductivity curves corresponding to different electromagnetic wave frequencies start to converge once again. This convergence indicates that while the photon energy is sufficient to interact with conduction electrons, its incremental influence on the transport characteristics becomes saturated. The system enters a regime where additional energy from the laser field no longer results in proportionally stronger coupling to the charge carriers.



a) b)

Figure 2. The variation of σ on Ω when $9.10^3 < \Omega < 10^4$ (a) and $10^6 < \Omega < 5.10^6$ (b).

Finally, at very high laser frequencies-on the order of $10^{14}\ s^{-1}$, Figure 3a shows that the conductivity stabilizes at approximately 0.56S/m, effectively becoming independent of the laser field. This saturation behavior suggests that the system has reached a limit where either all relevant transitions have been exhausted or the scattering mechanisms are no longer sensitive to further increases in photon energy. In such high-frequency regimes, electron dynamics may become dominated by other intrinsic factors such as phonon-limited scattering or interband transitions beyond the considered acoustic phonon interaction model.



a) b)

Figure 3. The variation of σ on Ω when Ω> 10^{14} (a) and The dependence of σ on F (b) .

Overall, these observations confirm that the laser field influences conductivity in a highly frequency-dependent manner. While negligible at low frequencies, the effect becomes significant at intermediate frequencies and eventually saturates at high frequencies, illustrating the nonlinear and nonmonotonic response of low-dimensional semiconductor systems to external optical perturbations.

Figure 3b presents the dependence of the electrical conductivity on the amplitude of the applied laser field. The results indicate that the amplitude exerts a significant influence on the transport properties of the system. As the amplitude increases, the conductivity exhibits a pronounced nonlinear rise, highlighting the strong field-induced enhancement of electron mobility. In the regime of low laser amplitudes, the three curves—corresponding to different values of the polarized electromagnetic wave frequency—almost completely overlap. This suggests that under weak laser excitation, the influence of the field on the electronic states is minimal, and thus the overall conductivity is not markedly altered.

However, as the laser amplitude increases beyond a certain threshold, the three curves begin to separate distinctly. This divergence implies that the laser field begins to play a dominant role in shaping the electron dynamics, and its interplay with the polarized electromagnetic wave becomes increasingly significant. The separation of the curves reflects the fact that the conductivity becomes sensitive not only to the laser amplitude but also to the frequency and presence of the electromagnetic wave, evidencing a coupled effect between the two external perturbations.

Physically, this behavior can be interpreted in terms of photon-induced transitions and electron–phonon scattering. A higher amplitude laser field corresponds to an increased number of incident photons, thereby raising the probability of photon–electron interactions. These interactions promote electronic transitions between quantized subbands or within the same subband (intraband transitions), effectively increasing the population of mobile carriers. Concurrently, the enhanced photon energy density facilitates a stronger modulation of the electron–phonon interaction landscape, particularly for acoustic phonons, which are the dominant scattering mechanism in this study.

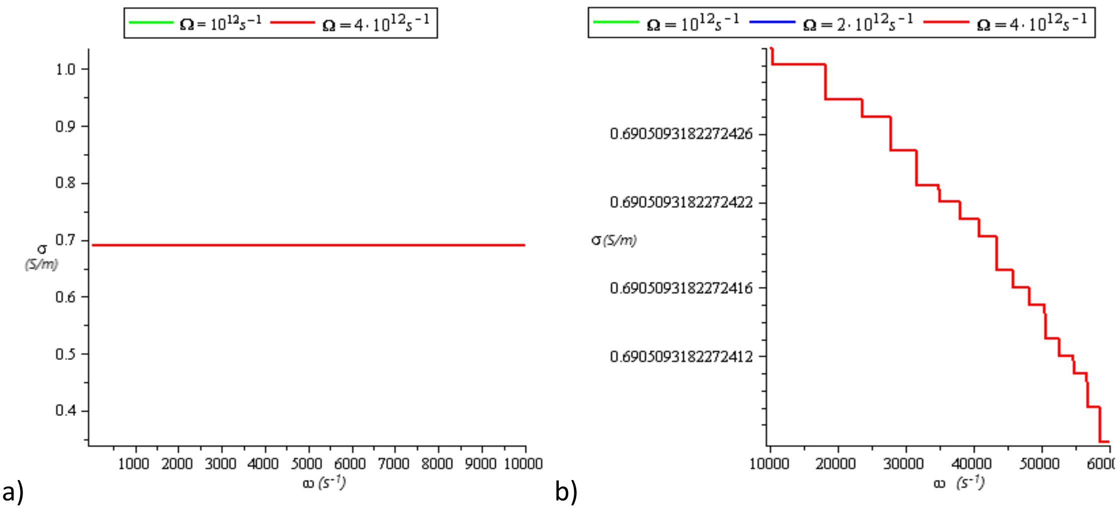


Figure 4. The variation of σ on ω when $\omega < 10^4$ (a) and $10^4 < \omega < 6.10^4$ (b).

Moreover, the external electromagnetic wave also contributes to this process. When its amplitude becomes sufficiently large, the associated electric field exerts a stronger force on the conduction electrons. This increased force can delocalize electrons further from their equilibrium positions, reducing their effective binding within the quantum well and enhancing their mobility. The cooperative action of the laser and electromagnetic fields thus leads to a synergistic increase in conductivity, especially at high field amplitudes. In summary, the amplitude of the laser field emerges as a critical parameter in determining the conductivity response of the system. At low amplitudes, its role is marginal, but at higher amplitudes, it not only significantly increases conductivity but also reveals a complex interplay with other external field parameters, underlining the importance of multi-field control in quantum-confined semiconductor systems.

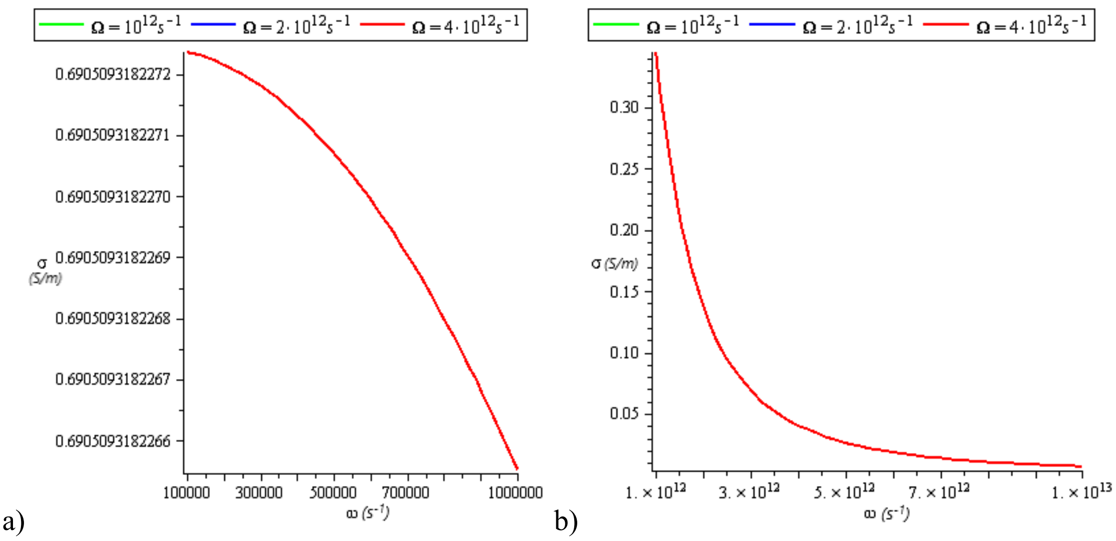


Figure 5. The variation of σ on ω when $10^5 < \omega < 10^6$ (a) and $\omega > 10^{12}$ (b).

Figure 4a presents the dependence of electrical conductivity on the frequency of the polarized electromagnetic field. At low frequencies (below 10^4 s^{-1}), the photon energy associated with the field is insufficient to induce noticeable modifications in electron dynamics. Consequently, the conductivity remains nearly constant, and the corresponding curves for different laser frequencies overlap closely. This observation confirms that in this regime, the electromagnetic field contributes negligibly to the modulation of transport properties.

As the frequency increases to the intermediate range (10^4 to $6*10^6$ s^{-1}), illustrated in Figure 4b, the conductivity begins to decrease gradually. However, the three curves still exhibit minimal deviation from one another, indicating that the influence of the polarized electromagnetic field remains weak compared to that of the laser field. The energy associated with the electromagnetic wave remains significantly lower, and thus insufficient to induce substantial changes in the mobility or scattering rate of conduction electrons.

A more noticeable variation emerges in the frequency window between 10^5 s^{-1} and 10^6 s^{-1}, as depicted in Figure 5a. Here, the conductivity exhibits a parabolic decline, suggesting the onset of photon-induced modifications to carrier dynamics. Nevertheless, the photon energy in this range remains relatively modest, and the associated influence on electron mobility, although visible, is still limited.

The most significant changes are observed in the high-frequency domain, particularly beyond 10^{12} s^{-1}, as shown in Figure 5b. In this range, the conductivity exhibits a sharp decrease as the frequency increases toward 10^{13} s^{-1}. This behavior corresponds to a regime in which the photon energy becomes comparable to, or exceeds, characteristic energy scales such as subband separations and phonon energies. As a result, the electromagnetic field begins to significantly perturb electron localization and scattering processes, leading to a marked reduction in conductivity.

However, once the frequency surpasses approximately 10^{13} s^{-1}, the rate of decrease in conductivity slows down, and the curves begin to flatten. This saturation effect indicates that the interaction between the electromagnetic field and the electron system reaches a limiting regime. Beyond this point, further increases in field frequency do not lead to proportionally stronger effects on transport properties, likely due to limits in the available electronic transition states or the onset of quantum coherence effects that stabilize electron dynamics.

TABLE 1. Some values of σ at key frequency points.

(Ω, σ)	(10^3, 0.5697273)	(10^6, 0.5697271)	(10^{12}, 0.5697270)	(10^{13}, 0.5697270)
(ω, σ)	(10^3, 0.69050931)	(10^6, 0.69050931)	(10^{12}, 0.35)	(10^{13}, 0.025)

Table 1 shows that the variation of conductivity with laser field frequency is quite small, while the variation of conductivity with electromagnetic wave field frequency is different. At small electromagnetic wave frequencies, the variation of σ with ω is small, but at large electromagnetic wave frequencies, σ decreases very rapidly as ω increases. From the comprehensive analysis of the frequency dependence, it is evident that electromagnetic waves with low to moderate frequencies interact only weakly with the confined electronic system, resulting in minimal changes to conductivity. This behavior aligns with theoretical expectations, as the photon energy at these frequencies is insufficient to perturb quantum-confined carriers effectively. In contrast, laser fields—with their highly coherent nature, high directionality, and relatively large photon energy and momentum—exert a far more pronounced influence on carrier transport. While both the laser and polarized electromagnetic fields are electromagnetic in nature, their differing energy scales and interaction mechanisms with phonons and electrons lead to markedly distinct impacts on the conductivity tensor of the quantum well system.

4. Conclusions

The study focuses on analyzing and discussing the influence of two external fields on the conductivity of a quantum well with infinite potential. The interaction that is mainly studied here is the interaction between electrons and acoustic phonons. The low-frequency electromagnetic wave field will not affect the conductivity of the system much, the interaction is only significant when this frequency is greater than $10^{12}s^{-1}$, at which time the conductivity in the quantum well decreases sharply. For the field of laser, its frequency has a very small impact on the conductivity of the quantum well, but its amplitude has a great effect on the conductivity, the conductivity increases as the amplitude increases. The influence of the two external fields is relatively different on the system, each of which has its own characteristics, so it has its own influence. This research has potential applications in semiconductor technology, where we control the conductivity of materials based on an external field, which helps to create conductive materials suitable for different uses.

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