SOME MEASURES TO FOSTER PROBLEM-SOLVING COMPETENCE IN MATHEMATICS FOR 10TH GRADE STUDENTS THROUGH TEACHING ALGEBRA KNOWLEDGE

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Abstract

Mathematics education contributes to the formation and development of students' key qualities, general abilities and mathematical abilities. In particular, the ability to solve mathematical problems is one of the core elements of mathematical ability, helping students develop key knowledge and skills and creating opportunities for students to experience and apply practical mathematics. The study proposes some measures to foster students' mathematical problem-solving capacity in teaching the topic "Algebra" (Math 10). Teaching practice shows that the proposed measures are appropriate and feasible, initially contributing to improving the quality of teaching and meeting the goals of educational innovation specified in the General Education Program math pass 2018.

Keywords: algebra, competency, grade 10 math, solving mathematical problems

1. Introduction

The 2018 General Education Mathematics Curriculum emphasizes that mathematical competence consists of five core components: mathematical reasoning and thinking skills; mathematical modeling competence; mathematical problem-solving competence; mathematical communication skills; and the ability to use mathematical tools and resources (Ministry of Education and Training, 2018). Therefore, developing students' mathematical problem-solving competence is considered a fundamental and essential objective.

For 10th-grade students, engaging with this topic remains relatively challenging, as they have had limited opportunities to explore and solve problems connected to real-life situations. Therefore, teachers need to implement appropriate instructional strategies to foster students' mathematical problem-solving competence. This paper presents the theoretical foundations and proposes several pedagogical measures aimed at developing students' problem-solving competence through the teaching of Algebra (Mathematics 10).

2. Literature Review

Numerous studies by scholars have addressed mathematical problem-solving competence. For instance, Ha Xuan Thanh (2017) discussed the teaching of mathematics

in high schools oriented towards developing students' competence in solving real-world problems through the integration and utilization of practical situations. Phan Anh Tai (2016) evaluated students' problem-solving abilities in the context of high school mathematics instruction. Nguyen Ngoc Ha and Nguyen Van Thai Binh (2020) focused on fostering mathematical problem-solving competence through the teaching of equations using the vector method in high schools. However, there has been limited research on cultivating students' mathematical problem-solving competence specifically through the teaching of algebraic content (Mathematics 10). Therefore, there remains a need for innovative approaches targeting specific student groups in the teaching of mathematics in order to enhance their problem-solving competence.

According to Nguyen Thuy Thuy Trang (2019), problem-solving competence reflects an individual's ability, either working independently or collaboratively, to think through problematic situations and to seek out and implement appropriate solutions. Tran Minh Man (2019) defines problem-solving competence as "an individual's ability to effectively apply cognitive processes, actions, attitudes, motivation, and emotions to address problems for which no predefined procedures, rules, or conventional solutions exist."

According to Phan Van Ly and Le Hoang Vinh (2022), instructional activities aimed at developing students' mathematical problem-solving competence include:

Activity 1: Designing learning experiences that help students identify and recognize problems that can be addressed using mathematics.

Activity 2: Guiding students to select and propose appropriate methods and solutions for problem solving.

Activity 3: Facilitating students' evaluation of the proposed solutions and their ability to generalize the approach to similar problems.

3. Methods

This study employs a theoretical research method to analyze, synthesize, and systematize relevant concepts and teaching activities from previously published scientific works related to the design of instructional situations in Algebra (Grade 10 Mathematics). In addition, a qualitative research approach is also incorporated through interviews with mathematics teachers at upper secondary schools in Binh Duong Province, serving as a basis for constructing measures to foster students' mathematical problem-solving competence.

4. Research Findings

4.1. Theoretical Foundations

4.1.1. The Concept of "Competence"

According to Nguyen Thi Thanh Thuy (2018), competence is understood as the ability to apply knowledge, skills, attitudes, and values to solve problems in life and to function effectively within a community. Weinert (2001) defines competence as cognitive abilities and skills, either innate or acquired, that enable individuals to solve problems arising in life. Competence also encompasses a readiness to act, motivation, willpower, and social responsibility in order to apply solutions effectively and responsibly in changing situations. According to Hoang Phe and colleagues (2008), competence refers to the

ability or inherent subjective and natural conditions to carry out a particular activity; it is a combination of psychological and physiological qualities that enable a person to perform a specific type of activity with high quality.

In this paper, we adopt the definition of "competence" as proposed by the Ministry of Education and Training (2018): Competence is a personal attribute that is formed and developed through inherent aptitudes as well as the process of learning and training. It enables individuals to effectively mobilize and integrate knowledge, skills, and personal qualities such as interest, belief, and willpower in order to successfully carry out a specific type of activity and achieve the desired outcomes under given conditions.

4.1.2. The Concept of "Problem Solving"

According to Bruder (2016), the process of problem solving should be at the core of general mathematics education; cognitive and reasoning aspects are of utmost importance, as they help students regulate their attitudes, emotions, and behaviors. Branford (1984) identified five components of the problem-solving process: identifying the problem, thoroughly investigating difficulties, proposing a solution, implementing the solution, and evaluating its effectiveness.

In this study, we define "problem solving" as the process of finding solutions to problematic situations encountered in practice. It is a creative process that begins with analyzing and identifying the problem, followed by generating and executing a solution. Finally, the solution is evaluated, and, if necessary, a more optimal solution is sought and applied.

4.1.3. Mathematical problem-solving competence

We define mathematical problem-solving competence as the ability to effectively solve a given mathematical problem by applying existing knowledge, experiences, and skills.

According to the Ministry of Education and Training (2018), high school students' mathematical problem-solving competence consists of the following components, each associated with specific indicators: (1) Identifying and recognizing the problem to be solved using mathematics: identifying problematic situations; collecting, organizing, interpreting, and assessing the reliability of information; and sharing understanding of the problem with others; (2) Selecting and proposing methods and solutions for problem solving: choosing and formulating appropriate approaches and procedures for solving the problem; (3) Applying relevant mathematical knowledge and skills (including tools and algorithms) to solve the problem: implementing and presenting the proposed solution; (4) Evaluating the proposed solution and generalizing it to similar problems: assessing the implemented solution, reflecting on its value, and generalizing the approach to other analogous problems.

4.2. Proposed measures to foster mathematical problem-solving competence in students through the teaching of algebra (Math 10)

4.2.1. Stimulating student motivation in Algebra lessons to foster mathematical problem-solving competence in grade 10 students

Purpose of the measure: Through motivational activities, students are encouraged to actively and proactively engage with new knowledge, thereby constructing understanding on their own. This measure offers numerous opportunities to develop components of mathematical problem-solving competence, particularly the ability to identify and recognize problems that need to be addressed using mathematics.

Implementation of the measure: To implement this measure, teachers introduce real-life, problem-based situations within the context of teaching Algebra. They guide students to actively, proactively, and creatively identify the core problems to be solved. Through this process, students construct knowledge, develop skills, and achieve the intended learning objectives.

Example 1: When teaching the concept of "Functions and Graphs" (Lesson 1 – Chapter III – Math 10), the teacher introduces the notion of a function through a weather forecast table for Ho Chi Minh City on May 1st, 2021, as follows:

The teacher raises a problem related to the relationship between temperature and time, prompting students to explore and recognize their dependency.

Scenario: Observe the temperature chart for Ho Chi Minh City on May 1st. Based on the data, how is temperature related to time?

Figure 4.1. Temperature on May 1st in Ho Chi Minh City (Source: Grade 10 Mathematics Textbook, Volume 1, Chan troi sang tao)

The teacher anticipates the following expected student responses after completing the assigned task:

• Each time interval corresponds to a specific temperature.

• Some time intervals may have the same temperature.

• As time changes, the temperature also varies accordingly.

This scenario is intended to help students understand the dependency between the two variables:temperature and time, based on the temperature chart. It serves as an initial step to stimulate students' interest and curiosity in exploring the concept of functions. From this, students are guided toward forming the definition of the concept "Functions and Graphs."

Example 2: When teaching the concept of "Systems of Linear Inequalities in Two Variables", the teacher guides students to approach the concept through the following real-life scenario:

Scenario: This year, an air-conditioning store plans to sell two types of air conditioners, two-way and one-way models, using an initial investment that must not exceed 1.2 billion VND.

	Air conditioners, two-way models	**Air conditioners, one-way models**
Purchase Price	20 million VND/unit	10 million VND/unit
Expected Profit	3,5 million VND/unit	2 million VND/unit

The store estimates that total market demand will not exceed 100 units for both types combined. If you were the store owner, how many units of each type would you invest in to maximize total profit?

(Source: Introductory problem, Grade 10 Mathematics Textbook, Ket noi tri thuc voi cuoc song series, Volume 1, p. 26)

The teacher organizes learning activities that enable students to form, recognize, and identify the concept of a system of linear inequalities in two variables through a series of guided questions.

Question 1: What information is provided in the problem statement?

Expected answer: The two types of air conditioners—two-way and one-way—with an initial investment not exceeding 1.2 billion VND, along with the purchase price and selling price of each type.

Question 2: What is the problem asking you to find?

Expected answer: To determine how many units of each type of air conditioner the store should invest in to maximize total profit.

Question 3: Let x and y represent the number of two-way and one-way air conditioners, respectively, that the store plans to purchase. Calculate the total investment required to purchase both types of air conditioners in terms of x and y.

The teacher guides students on how to express the total investment for purchasing both types of air conditioners as a function of x and y.

Expected answer: 20x +10y (in million VND).

Question 4: Since the total market demand does not exceed 100 units, what condition must x and y satisfy?

Expected answer: $x + y \leq 100$.

Question 5: Since the store owner's investment cannot exceed 1.2 billion VND, what condition must x and y satisfy?

Expected answer: $20x + 10y \leq 1200$ (in million VND)

The teacher guides students in determining the constraint on x and y based on the problem's requirements.

Through the above questions, students become motivated and interested in exploring new mathematical concepts. From there, the teacher leads them toward forming the definition of the concept "System of Linear Inequalities in Two Variables".

4.2.2. Organizing activities for solving real-world situations to foster mathematical problem-solving competence in students

Purpose of the measure: This measure aims to foster four key indicators of mathematical problem-solving competence: (1) The ability to identify and recognize problems that need to be addressed using mathematics; (2) The ability to select and propose appropriate methods and solutions for problem solving; (3) The ability to apply relevant mathematical knowledge and skills (including tools and algorithms) to solve the given problem; (4) The ability to evaluate the proposed solution and generalize it to similar problems.

Implementation of the measure: To implement this measure, teachers should design and conduct lessons that incorporate real-life situations related to Algebra topics in the Grade 10 curriculum. Within these situations, the teacher guides students in solving practical problems embedded in the mathematical context, with the goal of developing all four components of mathematical problem-solving competence.

Example 3: To foster students' mathematical problem-solving competence after the lesson "Systems of Linear Inequalities in Two Variables", the teacher presents the following real-world scenario for students to solve:

Situation: A family needs at least 900 units of protein and 400 units of lipids in their daily diet. One kilogram of beef contains 800 units of protein and 200 units of lipids. One kilogram of pork contains 600 units of protein and 400 units of lipids. The family plans to purchase no more than 1.6 kg of beef and 1.1 kg of pork. The price of one kilogram of beef is 160,000 VND, and one kilogram of pork is 110,000 VND.

Let x and y represent the number of kilograms of beef and pork, respectively, that the family needs to buy. Determine the values of x and y that minimize the total cost while still ensuring that the required amounts of protein and lipids are met.

(This scenario is adapted from Exercise 2.6, page 30, Grade 10 Mathematics Textbook, Ket noi tri thuc với cuoc song series, Volume 1.)

Activity 1: Organizing learning activities that help students identify and recognize problems that need to be solved using mathematics.

To help students select and propose appropriate methods and solutions for determining the number of kilograms of beef and pork the family should purchase, such that the total cost is minimized while still meeting the required amounts of protein and lipids, the teacher presents the following series of guiding questions:

Question 1: Which pieces of information in the problem can be used to formulate a system of linear inequalities in two variables?

Expected answer: A family needs at least 900 units of protein and 400 units of lipids in their daily diet. Each kilogram of beef contains 800 units of protein and 200 units of lipids. Each kilogram of pork contains 600 units of protein and 400 units of lipids. The price of one kilogram of beef is 160,000 VND, and one kilogram of pork is 110,000 VND. The family plans to buy no more than 1.6 kilograms of beef and 1.1 kilograms of pork.

Question 2: Which data in the problem can be used to formulate the expression F, representing the total amount of money the family has to pay when purchasing x kilograms of beef and y kilograms of pork?

Expected answer: The price of one kilogram of beef is 160,000 VND, and one kilogram of pork is 110,000 VND. The number of kilograms of beef is x, and the number of kilograms of pork is y.

Following this, the teacher provides guidance to students on how to represent the expression for the total cost the family must pay in terms of x and y.

Activity 2: Guiding students to select and propose appropriate methods and solutions for problem solving.

To help students apply their knowledge of systems of linear inequalities in two variables, the solution set of such systems, and the algebraic expression F, in order to solve the given problem, the teacher guides students through the following questions:

Question 1: Write the inequalities that represent the conditions of the problem as a system of linear inequalities in two variables.

Expected answer: From the questions and prompts in Activity 1, students can formulate the following linear inequalities and system of inequalities:

The total amount of money the family needs to pay is: $F(x, y) = 160x + 110y$ where x and y satisfy the following constraints:

$$\begin{cases} 0 \le x \le 1,6 \\ 0 \le y \le 1,1 \end{cases}.$$

The number of protein units required by the family is: $0,8.x + 0,6.y \ge 0,9$

$\Leftrightarrow 8x + 6y \ge 9$.

The number of lipid units required by the family is: $0,2.x + 0,4.y \ge 0,4 \Leftrightarrow x + 2y \ge 2$

The problem becomes: Find x, y satisfying the system of linear inequalities

$$\begin{cases} 0 \le x \le 1,6 \\ 0 \le y \le 1,1 \\ 8x + 6y \ge 9 \\ x + 2y \ge 2 \end{cases}$$

such that $F(x, y) = 160x + 110y$ *is minimized.*

Question 2: How can the solution region of the system of inequalities just formulated be determined?

Expected answer: Represent the solution set of each inequality on the coordinate plane; the solution region of the system is the intersection of the solution regions of all the inequalities in the system.

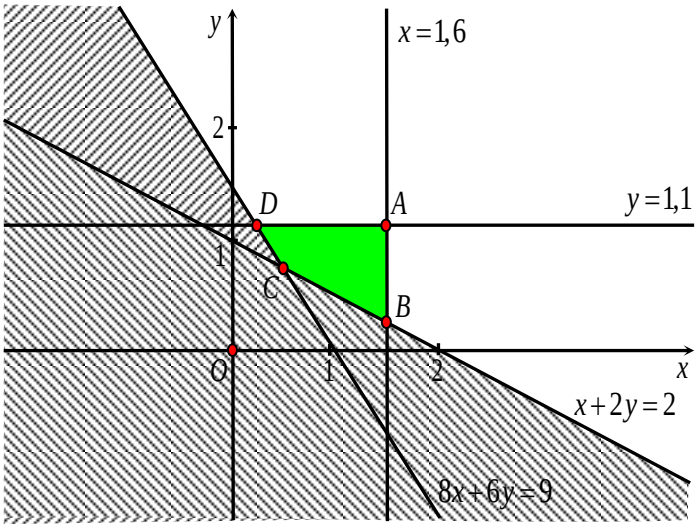

Figure 4.2. Graphical representation of the solution region of the system of inequalities in Example 3

The solution region of the system of inequalities is the quadrilateral ABCD with vertices at the coordinates: $A(1,6;1,1)$; $B(1,6;0,2)$; $C\left(0,6;0,7\right)$; $D(0,3;1,1)$.

Question 3: How can the total cost be minimized while still meeting the required amounts of protein and lipids in the diet?

Expected answer: Substitute the coordinates of the vertices of the solution region into the expression F to find the minimum value of F. Specifically: $F\left(A\right)=377$ thousand VND, $F\left(B\right)=278$ thousand VND, $F\left(C\right)=173$ thousand VND, $T\left(D\right)=169$ thousand VND.

It follows that the minimum value of F is 169 when x = 0.3 and y = 1.1.

Therefore, to minimize the total cost while still meeting the nutritional requirements, the family should purchase 0.3 kg of beef and 1.1 kg of pork.

Activity 3: Organizing activities for students to evaluate the proposed solution and generalize it to similar problems.

In this activity, the teacher facilitates tasks that help students explain the answers they previously provided, while also reinforcing the learned knowledge. This is done by posing a series of questions such as: How to formulate a system of linear inequalities in two variables? How to graph a linear equation in the form $ax+by=c$? How to determine the solution region of a linear inequality in two variables? How to find the value of an algebraic expression?.

5. Conclusion

Mathematical problem-solving competence is one of the fundamental mathematical competencies that needs to be nurtured in high school students. Developing this competence not only contributes to improving the quality of mathematics instruction but also helps students recognize the connection between mathematics and real-life contexts. This paper has proposed several measures for fostering mathematical problem-solving competence in students through the teaching of Algebra (Grade 10 Mathematics), with the aim of sparking students' curiosity and interest in learning, and encouraging them to actively explore and discover knowledge. Classroom practice shows that the proposed measures are appropriate and feasible. They have initially contributed to improving the quality of instruction and addressing the educational reform goals set forth in the 2018 General Education Mathematics Curriculum.

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